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An Enhanced Variational Mode Decomposition and Permutation Entropy Algorithm for Signal Denoising Applications

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Abstract:

Partial discharge (PD) detection is critical for assessing the insulation performance of power cables, as PD signals are often contaminated by periodic narrowband interference, white noise, and random pulse interference. Among these, narrowband interference poses significant challenges due to its large amplitude and high energy, which obscure the PD signals. Variational Mode Decomposition (VMD) has shown promise in denoising PD signals, but its performance heavily depends on the optimal selection of the number of modes (K). To address this issue, this paper proposes a novel method for determining the optimal K value based on the average orthogonal value, enabling accurate and intuitive selection to improve VMD decomposition performance. Additionally, permutation entropy is employed to reconstruct effective components and suppress periodic narrowband interference. Simulation and experimental results demonstrate that the proposed method effectively removes interference, minimizes PD signal waveform distortion, and achieves a high noise suppression ratio compared to conventional VMD and ensemble empirical mode decomposition methods. This approach provides a robust and reliable technique for PD signal denoising in power cable insulation assessment.

Keywords:

Partial Discharge; Mean Orthogonal Value; VMD; Permutation Entropy; De-noise.

1. Introduction

Partial discharge (PD) in cable insulation is an important cause of cable insulation deterioration. PD detection of power cables is an effective method to evaluate cable insulation performance [1]. In actual detection, PD signals are often affected by various types of interference and noise, so accurately extracting PD signals from noise staining signals becomes the primary task of power cable insulation state assessment [2]. According to the time-frequency characteristics of the interfered signals, they can be divided into periodic narrowband interference, white noise and random pulse interference [3]. Among them, the periodic narrowband interference has large amplitude and high energy, which often overwhelms PD signals [4]. At present, domestic and foreign scholars have done a lot of research on periodic narrowband interference and obtained some valuable results. The common methods are mainly fast Fourier transform [5], empirical mode decomposition (EMD) [6], variational mode decomposition (VMD) [7], etc. The spectral leakage effect of fast Fourier transform affects the noise reduction effect. EMD algorithm has the ability to process nonlinear and non-stationary signals, but it is prone to modal aliasing and end effect. VMD algorithm avoids some shortcomings of EMD algorithm, and can self-adaptively decompose different components in denoised PD signals into corresponding frequency bands. However, the decomposition effect of this method relies heavily on

the setting of the number of modes K value: too large setting of K value will cause over-decomposition, increase the amount of calculation and appear false components. If K value is set too small, underdecomposition will be caused, resulting in the loss of part of frequency information [8].

Aiming at the setting of K value of the optimal number of modes in VMD algorithm, literature [9] adopts the method of observing center frequency to determine K value. This method is simple to implement, but has strong subjectivity. In literature [10], the number of modal components obtained after EMD decomposition is used as the K value of VMD. Due to the mode aliasing phenomenon in EMD, the K value selected by this method is not accurate. Literature [11] proposes a double-threshold method to determine the optimal K value, but this method does not consider the influence of narrow-band interference.

According to the shortcomings of existing VMD denoising algorithms, an optimal K value determination method based on average orthogonal value is proposed in this paper, and the improved VMD algorithm is applied to the decomposition of PD signals. Secondly, the permutation entropy is used to select and reconstruct the effective components to remove the narrow-band interference. Simulation and experimental results show that the improved VMD and permutation entropy algorithm has strong noise suppression ability, and the extracted PD signal waveform distortion is small.

2. Basic Principle

2.1 Principle of VMD

VMD is a quasi-orthogonal and completely non-recursive signal decomposition estimation method, which can adaptively decompose signals into multiple modal components of limited bandwidth, and these modes are tightly around their corresponding center frequency [12].

(1) The intrinsic mode function is defined as a non-stationary amplitude-frequency modulation signal, and the Gaussian smoothness of its analytic signal is calculated. The constrained variational problem is obtained as follows:

$$\min_{\{u_k\},\{\omega_k\}}\left\{\sum_{k=1}^{K}\left\|\partial_t\left[\left(\delta(t)+\frac{j}{\pi t}\right)\times u_k(t)\right]\mathrm{e}^{-j\omega_k t}\right\|_2^2\right\}$$
$$\text{s.t.}\ \sum_k u_k(t)=f(t)$$

In Formula (1), K is the number of decomposed modes, and $\delta(t)$ is the unit impulse function.

(2) Use both the quadratic penalty term α and the Lagrange multiplier λ to make the problem unconstrained:

$$L\left(\{u_k\}\{\omega_k\},\lambda\right)=\alpha\sum_{k=1}^{K}\left\|\partial_t\left[\left[\left(\delta(t)+\frac{j}{\pi t}\right)\times u_k(t)\right]\mathrm{e}^{-j\omega_k t}\right]\right\|_2^2$$
$$+\left\|f(t)-\sum_{k=1}^{K}u_k(t)\right\|_2^2+\left\langle\lambda(t),f(t)-\sum_{k=1}^{K}u_k(t)\right\rangle$$

2.2 Improved VMD Algorithm

When the VMD algorithm is decomposing PD signals, it first needs to set the value of K. Due to the quasi-orthogonal relationship between the adjacent modal components obtained after VMD decomposition, if the improper selection of K value, the orthogonal relationship between the adjacent modal components will become worse [13]. The orthogonal value can be used to measure the orthogonality between two vectors, so as to reflect whether the selected K value is appropriate, but only by comparing the orthogonal values of two modes under different K values can not reflect the overall performance of VMD decomposition. However, the mean value is a statistic commonly used

in mathematics, which can take advantage of the characteristics of all data to represent the general level of the statistical object. Based on this, the index of mean orthogonal value is proposed as the basis for selecting the optimal K value in VMD algorithm. Mean orthogonal value (MOV) is the average sum of orthogonal values between each adjacent modal component. The value can reflect the overall orthogonality after VMD decomposition, and its calculation formula is as follows:

$$\mathrm{MOV} = \frac{1}{K-1}\sum_{i=1}^{K-1}\frac{C_i(t)*C_{i+1}(t)}{\|C_i(t)\|_2 * \|C_{i+1}\|_2}$$

2.3 Permutation Entropy

Permutation entropy is a method to detect randomness and dynamic mutation of time series. It has the advantages of simple calculation and strong robustness to noise [14]. Compared with PD signals, periodic narrowband interference shows stronger regularity in a certain frequency band, and the stronger the regularity of the signal, the smaller the permutation entropy of the signal. Taking advantage of this feature, the filtering of periodic narrowband interference can be realized by calculating the permutation entropy of the modal component and selecting the component with larger value for reconstruction. Set of length N one dimensional time series $X = \{x(i), i = 1, 2, 3... N\}$, the phase space reconstruction is carried out, and the following matrix is obtained:

$$Y = \begin{bmatrix} x(1) & x(1+\tau) & \cdots & x(1+(d-1)\tau) \\ x(2) & x(2+\tau) & \cdots & x(2+(d-1)\tau) \\ \vdots & \vdots & \vdots & \vdots \\ x(k) & x(k+\tau) & \cdots & x(k+(d-1)\tau) \end{bmatrix}$$

3. Noise Reduction Method based on Improved VMD and Permutation Entropy

Using the improved VMD and permutation entropy algorithm, the denoising steps of cable PD signal are as follows:

1. According to the actual needs, set the maximum number of modal component (IMF) N (N=10 in this paper), calculate the average orthogonal value under the corresponding number of modes, and determine the optimal number of modes K value.

2. VMD decomposition of the denoised PD signal is performed to obtain K modal components.

3. The permutation entropy of each modal component was calculated, and IMF component with larger value was selected for reconstruction to remove narrow-band interference.

4. Simulation Analysis

4.1 PD Signal Simulation

Literature [15] shows that PD signals of power cables actually detected are mostly in the form of attenuation oscillation. In this paper, double exponential attenuation oscillation pulse is used as the simulation model of PD signal, and the expression is as follows:

$$y(t) = A\left(\mathrm{e}^{-1.3t/\tau} - \mathrm{e}^{-2.2t/\tau}\right) sin\left(2\pi f_c t\right)$$

In the equation: A is signal amplitude; fc is the oscillation frequency and τ is the attenuation coefficient. The sampling frequency of the analog signal is set at 100 MHz, the signal amplitude is set at 5 mV, and the oscillation frequency is set at 2 MHz.

The attenuation coefficient can be set to different values, namely 0.1ns, 0.3ns and 0.5ns, respectively. The waveform of the simulation PD signal is shown in Figure 1:

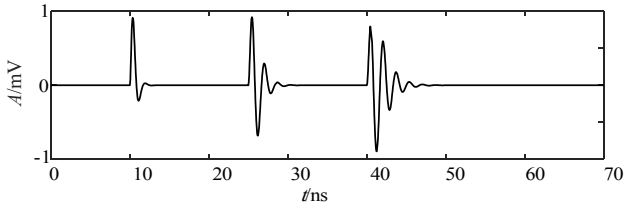


Figure 1. Original PD signal

Since periodic narrowband interference is mainly manifested as sine or cosine in the time domain, sinusoidal signals of different frequencies can be superimposed to simulate periodic narrowband narrowband interference. Periodic narrowband interference with frequencies of 1MHz, 6MHz and 8MHz and amplitudes of 0.1mV, 0.4mV and 0.3mV are added to the signal shown in Figure 1. As can be seen from Figure 2, PD signal has been completely submerged in noise signal.

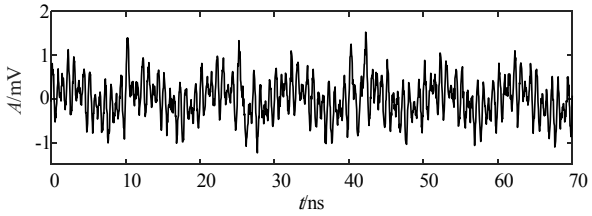


Figure 2. Noise PD signal

In order to quantitatively evaluate the degree of noise staining and the effect of noise reduction of PD signals, three indicators are introduced: noise suppression ratio (NRR), root mean square error (RMSE) and signal-to-noise ratio (SNR). SNR and NRR respectively reflect the noise content and effective signal prominence in signals, and the larger the value, the better. RMSE is used to measure the deviation between the observed value and the real value. The closer it is to 0, the smaller the deviation is. Through calculation, the SNR of PD signal with dyed noise as shown in Figure 2 is -14.12dB

4.2 Simulation Signal Noise Reduction Analysis

Firstly, the optimal K of VMD decomposition was selected based on the MOV method, and the MOV under different K values was shown in Figure 3.

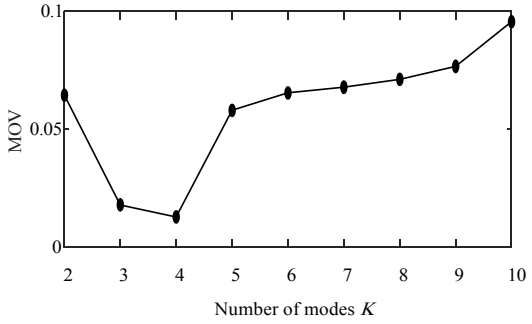


Figure 3. Line chart of mean orthogonal values under different number modes

The smaller the MOV, the better the VMD decomposition performance. As can be seen from Figure 3, the MOV is the minimum when the number of modes is 4. Therefore, this paper chooses K equal to 4 as the optimal mode decomposition number of VMD. The VMD decomposition of PD signal is carried out. As can be seen from the decomposition results in Figure 4, each frequency component in the denoised PD signal is well decomposed into different modal components, among which the IMF2 component contains the main information of PD signal.

Since the PD signal is composed of periodic narrowband interference white noise, the permutation entropy is used to filter the periodic narrowband interference first. The permutation entropy of IMF1~ IMF4 was calculated and normalized, and the results were shown in Table 1 below

Table 1. Permutation entropy of IMF components

IMF	1	2	3	4
H_P	0.243	0.712	0.322	0.411

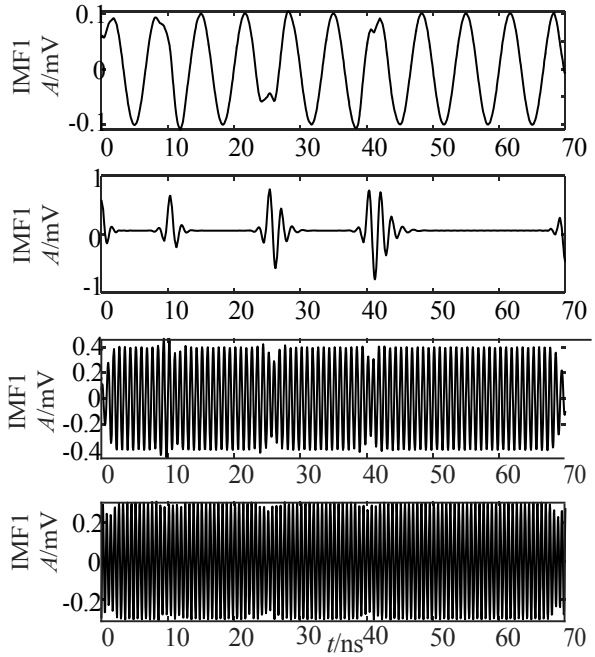


Figure 4. Line chart of mean orthogonal values under different number mode

The entropy value reflects the randomness of the signal. The periodic narrow-band interference has strong regularity and relatively low entropy. By changing the signal parameters for several simulation analyses, the H_P value greater than 0.5 is chosen as the judgment criterion for effective reconstruction component. It can be seen from the data in Table 1 that the entropy values of IMF1, IMF3 and IMF4 are all lower than the set threshold, indicating that the periodic narrowband interference is mainly concentrated in these three modes and is abandoned. The entropy of IMF2 is relatively large, so this component is used as the reconstructed PD signal. PD waveform after noise reduction is shown in Figure 5. It can be seen from Figure 5 that the original PD signal has been restored to a large extent, and the amplitude and waveform have not changed significantly. The calculated signal-to-noise ratio is 11.44dB after noise reduction. Compared with -14.12dB before noise reduction, the periodic narrowband interference has been significantly suppressed.

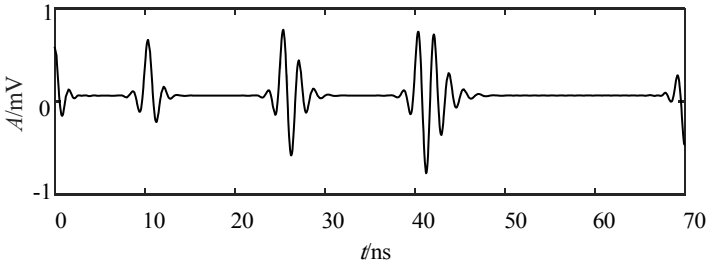


Figure 5. Noise reduction results of improved VMD and permutation entropy method

To comprehensively evaluate the noise reduction performance of the proposed method, a comparison was conducted with two widely used techniques: Variational Mode Decomposition (VMD) combined

with permutation entropy and the Empirical Mode Decomposition (EEMD) method. In the proposed method, VMD is first used to decompose the noisy partial discharge (PD) signal into intrinsic mode functions (IMFs), where the optimal number of modes (K value) is determined using the observation center frequency method. Permutation entropy is then applied to evaluate the significance of each IMF, ensuring that only the effective components containing meaningful signal features are retained while noise-dominated IMFs are discarded. The remaining components are reconstructed, effectively filtering out periodic narrowband interference and achieving robust noise reduction. In contrast, the EEMD method adaptively decomposes the PD signal into IMFs, and the kurtosis criterion is used to identify and retain significant components for reconstruction. The denoising results, presented in Figures 6 and 7, along with the evaluation metrics in Table 2, demonstrate that the proposed method achieves superior noise reduction performance compared to EEMD, with higher Signal-to-Noise Ratio (SNR), Normalized Cross-Correlation (NCC), and Noise Reduction Ratio (NRR) values, indicating its effectiveness in suppressing noise while preserving the critical characteristics of the PD signal.

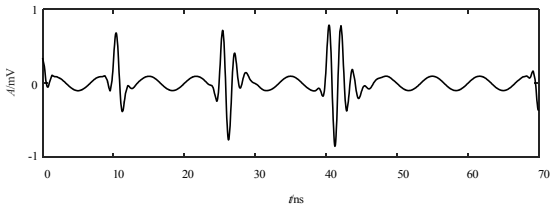


Figure 6. Noise reduction results of VMD and permutation entropy method

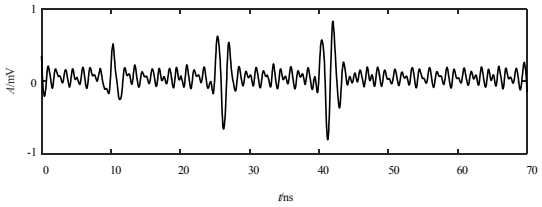


Figure 7. Noise reduction results of EEMD

SNR, NCC and NRR of the three methods were calculated respectively, and the results were shown in Table 2.

Table 2. Evaluation parameters of three methods after noise reduction

Evaluation Index	Improved VMD and Permutation Entropy Algorithm	VMD and Permutation Entropy Algorithm	EEMD Algorithm
SNR	11.436	9.884	7.561
NCC	0.963	0.915	0.812
NRR	12.228	10.604	9.511

By comparing Figure. 5, Figure. 6, Figure. 7 and Table 2, we can see:

(1) Although VMD and permutation entropy method, EEMD method can restore PD signal to a certain extent, the waveform after noise reduction all contains certain noise components, resulting in a certain degree of distortion. The main reason for waveform distortion after VMD and permutation entropy method is the improper selection of K value based on observation center frequency method, which leads to poor decomposition performance of VMD and finally affects the noise reduction effect. The distortion of EEMD waveform is mainly due to the mode aliasing phenomenon when EEMD algorithm decomposes PD signal.

(2) Improved VMD and permutation entropy method because the method based on average orthogonal value is used to determine the correct number of decomposition modes for VMD algorithm, so that the decomposition results are optimized, and it is obtained that PD signal after noise reduction has a higher signal-to-noise ratio, which is closer to the original signal waveform, and at the same time, the waveform characteristics of PD signal are better preserved

5. Conclusion

In order to effectively suppress the periodic narrowband interference in partial discharge detection of power cables, a noise reduction method based on improved VMD and permutation entropy is proposed in this paper, and the noise reduction effect of this method is tested by simulation PD signals. Aiming at the problem that the improper value of the optimal number of modes in VMD algorithm is easy to lead to mode aliasing and frequency information missing in decomposition, a method of mode number selection based on average orthogonal value is proposed, which can intuitively and accurately select the optimal number of modes, and thus improve the decomposition performance of VMD algorithm. The periodic narrow-band interference is suppressed by calculating the permutation entropy of the modal component and selecting the component with larger value for reconstruction. The simulation and experimental results show that, compared with the conventional variational mode decomposition method and ensemble empirical mode decomposition method, the proposed algorithm can effectively remove the periodic narrowband interference in PD signal during the insulation detection of power cable, and the waveform distortion of PD signal after noise reduction is small and the noise suppression ratio is high, which has certain application value.

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